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# **Data Fusion and Sensor Management**

Kaouthar Benameur

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**Technical Report**  
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Her Majesty the Queen as represented by the Minister of National Defence, 2002

Sa majesté la reine, représentée par le ministre de la Défense nationale, 2002

## Abstract

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This report addresses the problem of data fusion and sensor management based on a synergistic use of the information provided by multiple sensors. In the first studied case, we deal with the problem of measurement strategy computation for a passive receiver. The basic problem is then to compute an optimal policy, during a specified observation time interval so that a prediction accuracy is optimized. It is shown that the optimal measurement policy can be precomputed before the measurements actually occur. The second case describes research work on the selection of a strategy of measurements for an active system and a passive one. The approach is based on selecting at each instant of time, a set of measurements provided by one or more sensors. Each sensor measurement has an associated cost. The basic problem is then to select an optimal measurement policy, during a specified receding horizon observation interval, so that a weighted combination of prediction accuracy and observation cost is optimized.

## Résumé

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Ce rapport adresse le problème de fusion de données et de gestion des capteurs en se basant sur la l'utilisation synergique de l'information provenant de différents capteurs. Dans le premier cas étudié, on aborde le problème du calcul d'une stratégie de mesures pour un capteur passif. Le problème est alors de calculer une stratégie optimale durant un intervalle d'observation donné de façon à optimiser la précision des estimées. On montre qu'une stratégie de mesures peut être calculée avant de faire les mesures. Le deuxième cas étudié présente une stratégie de selection de mesures pour deux systèmes, un actif et un passif. L'approche se base sur la selection à chaque instant, d'un ensemble de mesures, provenant de un ou plusieurs capteurs. A chaque mesure est associé un coût. Le problème est alors de choisir les mesures durant un intervalle d'observation fuyant tel qu'une combinaison pondérée de la précision des estimées et du coût d'observation est optimisée.

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## Executive summary

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This report addresses the problem of data fusion and sensor management based on a synergistic use of the information provided by multiple sensors. In the studied cases, we formalize a probabilistic framework for information gathering by defining three components: geometric models, sensor observation models and models for prior information. Geometric models consist mainly of the target model, which describes the structure of the rigid body in a certain fixed coordinate system. Sensor models mathematically describe the mapping between the rigid body coordinates and the statistically corrupted sensor observations. Prior information is encoded in a structured environment assumption, as geometric and dynamic models are assumed known.

Within this framework and using an optimization criterion that relates the task at hand to the information gathering process, we show that the performance of a designed system depends highly on an interplay between the used sensors and the sensor manager. This motivates the development of adaptive strategies for observations. These strategies account for the uncertainties in sensors and dynamic models as well as the task description.

For the task of Target Motion Analysis (TMA), the adaptive strategy is based on a procedure to choose the sensors actions which yield the best estimation performance for a receding observation horizon. We derive and discuss the optimization approach; then show how to compute it efficiently, and demonstrate some of its properties through two studied cases. The first case is based on the feedback interconnection between two dynamical systems: First, a state estimator where the observation equation is a nonlinear map between the state variable of the target and the receiving system. This mapping depends on some external variables and on the relative position of the sensors with respect to the target. Second, a controller of the receiving system with the objective of minimizing some function of the covariance of the state estimation error.

The second case deals with the selection of a strategy of measurements for an active system and a passive one. The approach is based on selecting, at each instant of time, a set of measurements provided by one or more sensors. Each sensor measurement has an associated cost. The basic problem is then to select an optimal measurement policy, during a specified receding horizon observation interval, so that a weighted combination of prediction accuracy and observation cost is optimized. Numerical results for second-order system with active and passive measurements are presented.

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## Sommaire

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Ce rapport aborde le problème de la définition de stratégies de mesures de plusieurs capteurs dont l'objectif principal est la détermination du mouvement d'une cible dans un environnement donné. Une des contributions de ce rapport est l'introduction des mécanismes adaptatifs au sein de la fusion de données. Ces mécanismes adaptatifs touchent le nombre de capteurs utilisés, leurs types, ainsi que les paramètres externes de chaque capteur sélectionné: sa position et sa trajectoire. Pour analyser le mouvement d'une cible, la stratégie adaptative est basée sur les choix du capteur et de son action qui permettent la meilleure performance d'estimation sur un intervalle d'observation fuyant.

Dans ce rapport, le premier cas étudié, se base sur l'inter-connection en feedback de deux systèmes dynamiques: Premièrement, l'estimateur d'état où l'équation d'observation est nonlinéaire et présente la projection du vecteur d'état dans l'espace mesures. Cette équation est une fonction du vecteur d'état et de la position relative du capteur par rapport à la cible. Deuxièmement, le contrôleur du capteur dont l'objectif est la minimization d'une fonction de la covariance de l'erreur sur les estimées. Le deuxième cas étudié aborde la sélection d'une stratégie de mesures pour deux systèmes, l'un actif et l'autre passif. L'approche est basée sur la sélection, à chaque instant, d'un ensemble de mesures provenant d'un ou plusieurs capteurs. Le problème de base est alors de choisir une stratégie de mesures optimale sur un intervalle d'observation fuyant qui permet d'optimiser la précision de la prédiction de l'état de la cible et de minimiser le coût de l'observation. Les résultats numériques sont présentés dans le cas d'un système de second ordre avec des mesures actives et passives.

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# 1. Introduction

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The world is becoming increasingly information-centric and often systems are overwhelmed by the amount of the data, giving rise to the problem of "information overload". Therefore, the need exists to develop cost-effective multi-source information systems that require adaptive methods for specifying data fusion processing and control functions, and associated data bases.

These main issues in sensor data fusion have mostly been addressed separately, sometimes based on well-founded theories and sometimes in an ad hoc manner and in the context of specific systems and architectures. Data fusion as defined in this report, refers to the "synergistic use of the information provided by multiple sensory devices to assist in the accomplishment of a task by a system"[1]. Sensor synergy can be described as the organization, coordination and management of sensors and the combination of the information they provide such that their overall operation is complementary and non-conflicting given the sensing needs of the system. This synergistic operation between sensors has also been termed sensor management, sensor coordination and sensor planning and control [7].

## 1.1 Resource Management and adaptive data fusion

Different sensors may have the capability to measure different features of the same target. For example, a radar has range, range rate, and moderate angle measurements capabilities, while an ESM, or IRST, has angle measurement capabilities. Differences in sensor ID measuring capabilities are also prevalent. The goal of data fusion is to combine the varied sensor data to achieve a better overall picture of the environment. In this process, resource management, constrained here to the sensors, plays the essential role of coordinating the data collection processes of the various sensors to support the overall goal. Thus, for example, resource management may respond to a tactical need for a more precise ID or more range accuracy by cueing the radar to support a track that was primarily maintained by the ESM. Sensor management is therefore an optimization of the measurement process to achieve an overall goal.

This natural coupling between data fusion and resource management is recognized in the literature [27]. Resource management is a key element of the data fusion process where it is essential that these sensors operate synergistically. Resource management in a multisensor system must operate so that full advantage is taken of the strengths of each sensor. Sensor management can accomplish the coordination between sensors through cooperative reinforcement, emission control, sensor cueing, situation assessment, and adaptive behavior in varying sensing environments where the ability of a sensor to collect information can be strongly influenced by the environmental conditions. These conditions can include weather, electronic countermeasures, clutter,...etc. In emission control, we consider two basic sensor categories: (1) passive

and (2) active. Passive sensors collect information on elements in the environment by measuring energy which the targets emit or which reflects off them from sources in the environment other than the sensor itself. Thus, passive sensors can covertly collect information about an element in the environment. Active sensors on the other hand actively emit a signal in an attempt to reflect it off an element in the environment. Unfortunately, it is generally true that active sensors can provide more information than passive sensors. Therefore, in a covert operation, there is a significant advantage in minimizing the use of active sensors. Sensor cueing may improve the response time of a sensor by cueing it with information derived from another sensor. In this report, we consider a radar as an active sensor measuring the range and the angle and an ESM as a passive sensor. It generally measures only angle. Because the operation of the radar betrays the presence of ownship, sensor management should attempt to synergistically employ the radar and the ESM so as to minimize active radar radiation and optimize the quality of the track.

Steinberg [28] gave the following more general definition to resource management: A resource management process is one that combines multiple available actions over time to maximize some objective function. Such a process must contend with uncertainty in the current situational state and in the predictive consequences of any candidate action. A resource management process will:

- develop candidate response plans to estimated world states;
- estimate the effects of candidate actions on mission objectives;
- identify conflicts for resource use; and
- resolve conflicts based on the estimated impact on mission attainment.

Coordination across multiple sensors has been the focus of many research activities mainly in the multitarget/multisensor tracking context where a recent trend is the availability of multiple sensing modalities that differ in such crucial measures as detection, estimation, geographical coverage and cost of operation. Considering a target tracking and identification system, sensor management attempts to achieve the overall system optimization by checking target tracking and identification performance relative to certain criteria and generating a feedback control signal to the sensors. Nash recognized that the Kalman filter covariance matrix ( or equivalently the information matrix) is one primary measure of a tracking system's current information state. References present Kalman filter-based sensor allocation methods. It is interesting to note that in the literature, we find two views about what needs to be managed in managing sensors: The parameter view of managing sensors requires the sensor manager to directly control each degree of freedom of the sensor. The mode view greatly simplifies the sensor management decision making. This simplification is a natural consequence of mode design. In a non-sensor-managed system, modes are simply a careful trade over the degrees of freedom which optimize a priori performance criteria.

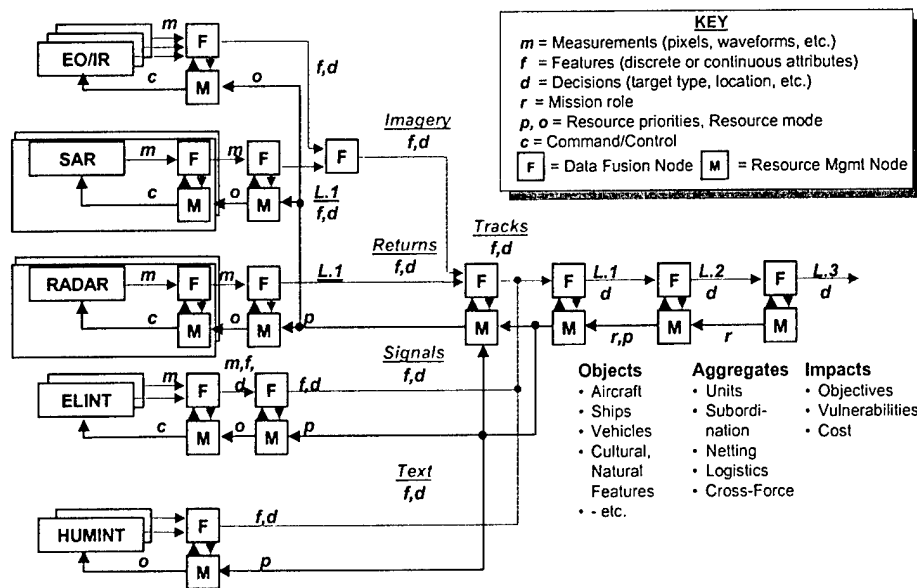


Figure 1: Integrated data fusion/resource management tree (Steinberg 1998)

Adaptive data fusion is an important avenue of research by which the selection of data to be processed and of the processing techniques to be applied is determined by a system's resource management process during run-time. Referring to Steinberg's data fusion processing diagram, when the data fusion process is partitioned into multiple processing nodes, and the process is represented via a data fusion tree as illustrated in figure 1 then the data fusion tree and nodes are constructed adaptively, based on the system's assessed current information state and the predicted effectiveness of available techniques to move to a desired information state. Significant work in this area was conducted under the US DARPA Dynamic Multi-User Information Fusion (DMIF) project. Data Fusion Engineering Guidelines that were developed in this project recommend an architecture concept that represents data fusion systems as networks of processing nodes, each node having the structure shown in figure 2.

Figure 3 shows the concept of adaptive data fusion as defined by Steinberg. In his definition, he extended the concept of adaptive sensor fusion to include the coordinated use of sensors, communications, processing, and response systems (weapons, countermeasures). Traditional data fusion involves the feed-back loop where estimates of the observed situation are used along with prior models to interpret new data.

The adaptive sensor fusion concept as defined by Steinberg includes two more feedback loops: resources are allocated based on the current estimated state and the desired state. Additionally, the system refines its library of models, target and background models as well as models of resource performance as their performance is assessed in mission (

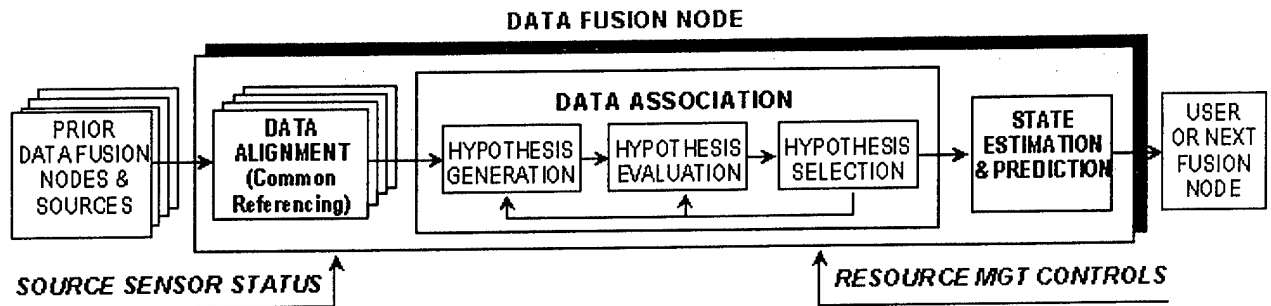


Figure 2: Data fusion processing node (Steinberg 1998)

figure 3).

An adaptive data fusion system will characterize off-normal measurements in terms of four components:

1. Random process noise affecting the observations of an individual target entity
2. Random process noise affecting entire classes of entities; e.g., random behavioral or design variability
3. Deterministic change in the individual target entity; for example a manoeuvre, a signature change (due to a damage or a temperature change), and
4. Deterministic change affecting entire classes of entities (doctrinal or design changes).

The problem of resource allocation has a long history mainly with radar. In the scheduling of radar resources, the radar sensor manager, considered as an autonomous system till recently, has to determine a set of task priorities based on the radar track file and the internal radar information. The major task categories are (1) update and target identification for existing tracks, and (2) search for new targets. Considering an agile beam radar, three interrelated parameter choices are involved in the efficient update of existing target tracks. The first is the choice of revisit time, which is directly related to the track prediction error relative to the radar beamwidth. Second, once a track update is allocated, the SNR, which is directly proportional to Time On Target (TOT), must be chosen. Finally, the detection threshold, with resulting  $P_{FA}$ , must be chosen. A modern efficient allocation algorithm must direct the agile beam radar among the options to search for new targets or to update the existing tracks or to remain covert. This decision is based on many factors but basically is mission dependent. Thus, the basic issue in the design of a scheduling algorithm is the manner in which these alternative tasks are assigned Figures of Merit (FOM). The two basic methods that

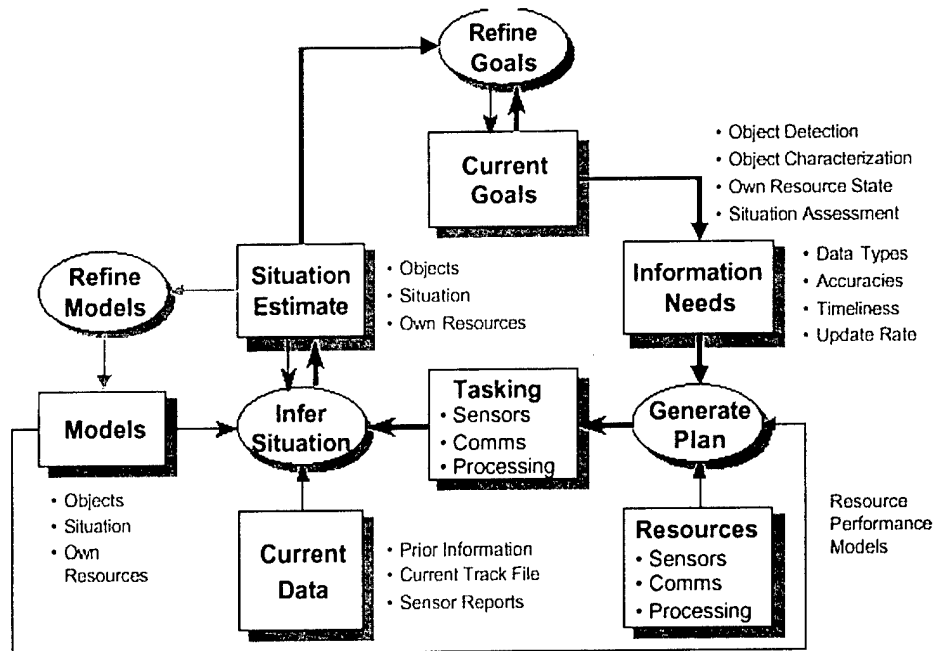


Figure 3: Adaptive information exploitation (Steinberg 1998)

have been proposed for determining FOM are through the utility theory and expert system approaches.

Several FOM are used in the design and performance evaluation of sensor managers [5]. These FOM are chosen as a function of the task at hand:

- Target acquisition FOM: the two basic types of target acquisition performance metrics are instantaneous and cumulative. The instantaneous metrics measure the ability of a sensor system to recognize the existence of a target during a predefined short period of time. This metric can be defined as the probability of detection as a function of range. A common FOM is the range at which a target will be detected on 50% of the trials. The cumulative metrics describe a sensor's ability to recognize the existence of a target during the course of a predefined engagement geometry. These metrics measure the range by which the sensor system has a fixed probability of detecting the target. These metrics only have meaning in terms of a specified target geometry.
- Track acquisition FOM: One typical FOM which is reminiscent of the acquisition FOM is the range at which the target has a 90% probability of confirmed track. Other important metrics are based on the sequential probability ratio test.
- Kinematic accuracy FOM: The primary kinematic accuracy FOM is the standard deviation of the tracking error. The calculation of this FOM is part of the estimation process.

- Track purity FOM: typical FOM are probability of correct association, probability of correct decision, and generalized covariance analysis.
- Weapon control FOM: They are dependent on the weapon specifications.
- ID ambiguity FOM: ID ambiguity FOM are the attribute fusion counterpart to covariances for kinematic fusion.

Covariance matrix of the track filter has been conveniently used as a criteria for the optimization of measurements scheduling. LeCadre [10] considered different norms of the Fisher information matrix. In his research, he analyzed performance for optimal scheduling of the multiple estimation modes for a non-linear system with a focus on non-linear effects in Target Motion Analysis (TMA) and global optimization. Expected discrimination gain has been considered as a criteria for this optimization problem mainly by Kastella [18], and Nimier [15]. This criteria is a measure of sensor effectiveness that has been used in a wide variety of model applications including multisensor/multitarget assignment problems, minimizing error correlation between close targets, and single and multisensor detection/classification problems. Discrimination is related to the notions of information and entropy in probability distributions. It measures the relative increase in information between two probability distributions.

However, before defining an FOM for the design and performance evaluation of managed sensors, there is a need to identify what is manageable i.e. parameters to vary to optimize some FOM. The application of sensor management involves the selection of the sensors and of all parameters that define the operation of each sensor. The multi-function radar is perhaps the sensor with the most degrees of freedom. Full management of this sensor includes general command categories of where to point, how to scan, waveform to transmit, and processing directives. Each of these command categories is specified by a number of parameters. For example the waveform selection involves frequency, PRF, length of coherent integration, and total time on target. In light of the complexity issue and the trade off between the parameters, we will explore a sensor management design that does not attempt to trade over very detailed parameters of sensor operation. Nevertheless, this research presents two comprehensive management cases where the benefits of fusion and management are evident. In the first case, we consider that the position of the passive sensor as a manageable parameter. In the second case, we consider a coordination problem between a passive sensor and an active one.

This report is organized as follows. After the introduction which provided an overview of the connections between data fusion, resource management, and adaptive data fusion, section 2 provides a detailed description of an optimization approach in computing the position of a passive sensor. Our objective is to estimate the kinematics of an emitting source. In section 3, the optimization problem is one of assigning sensors to a target in such a way that the error of the track prediction is minimized. This

section presents details on the optimization scheme used to compute the optimum assignment of sensors to target. Section 4 presents some suggestions for further work

## **2. Optimal ESM location for emitter tracking**

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The basic problem of Target Motion Analysis is to estimate the trajectory of an emitting source from noise corrupted sensor data. Suitable data are, in principle, all measurements which are functions of the target space. A particular interesting case arises when all measurements are derived from a single moving observer. Although a variety of techniques are available, a particular good method relies on using Doppler-shifted frequency measurements in addition to bearing measurements. This method was described and analyzed in a two-dimensional spatial setting by Becker[22], and in a three-dimensional setting by Fowler [23]. It was shown that the combined measurements produced significantly better locations due to the synergy gained from the non-alignment of the frequency and bearing error ellipses. In this work the bearing angles and the Doppler-shifted emitter frequency are considered.

Tracking the source target based on measurements collected by a fixed or a moving observer is a classical problem in the field of nonlinear estimation. In the tracking problem from angle and frequency measurements, an observer maneuver is necessary to ensure the observability of the emitter (uniqueness of the solution). However even with the observability requirement fulfilled, measurements are always corrupted by errors and the accuracy of the estimates may strongly depend on the maneuver of the observer. In this study we consider the information gathering as a dynamic process that responds to more than the tracking condition. In fact, in the proposed approach we try to integrate and coordinate between tracking and an accuracy criterion deduced from the Fisher Information Matrix (FIM) by considering the objective of minimizing some function of the covariance of the predicted state vector. In the proposed approach, the effect of measurements uncertainties are indirectly incorporated in the cost function.

Our problem presents some similarities to the localization problem. However, our main objective is the determination of the whole source trajectory instead of its detection at a fixed final time. To compensate for this intrinsic difficulty, we assume that the source is



already detected and we have measurements of its bearing and Doppler-shift frequency. It follows that the function to maximize is not the probability of detection during a given time, but the information relative to the source trajectory which can be inferred from the measurements and the observer trajectory. This information is mainly dependent on the observer trajectory.

## 2.1 Problem Formulation

To establish a dynamic model for the target motion, the following assumption is made in advance: our purpose is to estimate, rather than regulate, the target motion, so the resulting model is not necessarily controllable. The dynamic model for the target is assumed to be

$$(1) \quad \dot{x}(t) = f(x(t)) + \xi(t) \quad x(0) = x_0$$

It is a time varying system describing the position and the motion of the object in an inertial frame as well as the transmitted frequency, treated as an additional state variable. We assume that the system state vector  $x(t)$  is an  $n$  dimensional column vector. The system driving noise  $\xi(t)$  is assumed to be a Gaussian white noise  $N(0, Q)$  with zero mean and a fixed covariance matrix. The initial state vector  $x_0$  is modelled as a random vector with known mean,  $\hat{x}_0$ , and known covariance  $P_0$ . The dynamic model of the target motion may be as simple as  $\dot{x}(t) = Ax(t) + \xi(t)$  where  $A$  is known and as complex as a model for a ballistic missile.

The receiver model combines 3D geometric models of the scene with the laws of electromagnetic waves in order to describe the position of the target as a function of spatial positions and orientations of the target and of the receiver in an inertial frame in addition to describing the Doppler shift of the emitter frequency due to the relative motion between the sensor and the emitter.

The transformation between the inertial frame and the sensor frame induces nonlinear equations. The size of the measurements vector depends on the number of targets in view. For simplicity, we assume one source target in the field of view of the observer, the vector of measurement equations for the target can be written as:

$$(2) \quad y(t) = h(x(t), {}^iP_r) + \nu(t)$$

${}^iP_r$  defines the state vector of the sensor, limited in the case of a constant speed observer to the position of the observer in an inertial frame.  $\nu(t)$  is the measurements noise, defined as a Gaussian white noise with zero mean and constant, positive definite, covariance  $R(t)$ . In this study, we consider that the purpose of the sensor observations

during the time interval  $t_0 \leq t \leq T$ , is the estimation of the source trajectory and frequency. With the above assumptions and notations, the estimation will be carried out within the framework of Bayesian decision theory. Considering this probabilistic description of our estimate is particularly useful in the context of an active sensor. In such a system, new information is continually being acquired due to either observer or target motion or both, and estimates are continually being updated. A useful formalism for modelling such a system is the Kalman filter. One of the most compelling arguments for using a Bayesian framework is its mathematical clarity and elegance. Furthermore we believe that:

- In the situations we are considering, some prior information is available.
- We have a well justified basis for the chosen performance criteria.
- It defines a powerful tool to maximize the speed of convergence of the estimates to true values by application of advance knowledge.

For this and because of the nonlinearities of the models, we consider an extended Kalman filter to derive the estimates. The basic idea of the extended Kalman filter [24] is to relinearize the sensor and dynamic models about each predicted estimate once it has been computed. As soon as a new state estimate is made, a new and better reference state trajectory is incorporated into the estimation process. Thus allowing the validity of the assumption that deviations from the nominal trajectory are small enough to allow linear perturbation techniques. Under the assumption of continuous time process and continuous time measurements:

- The continuous nonlinear system dynamics are

$$(3) \quad \dot{x}(t) = f(x(t)) + \xi(t)$$

where

$$(4) \quad E\{\xi(t)\} = 0$$

$$(5) \quad E\{\xi(t)\xi^T(t')\} = Q\delta(t - t')$$

$$(6) \quad E\{x(t_0)\} = \hat{x}_0$$

$$(7) \quad E\{(x(t_0) - \hat{x}_0)(x(t_0) - \hat{x}_0)^T\} = P_0$$

$$(8) \quad E\{(x(t_0) - \hat{x}_0)\xi^T(t)\} = 0$$

- The continuous nonlinear measurement system is

$$(9) \quad y(t) = h(x(t), t) + \nu(t)$$

where

$$(10) \quad E\{\nu(t)\} = 0$$

$$(11) \quad E\{\nu(t)\nu^T(t')\} = R(t)\delta(t - t')$$

$$(12) \quad E\{(x(t_0) - \hat{x}_0)\nu^T(t)\} = 0$$

$$(13) \quad E\{\xi(t)\nu^T(t')\} = 0$$

The extended Kalman filter cycle is given by the following equations [6]

$$(14) \quad \begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t)) + PH[x, {}^iP_r]^T R^{-1}(t)[y(t) - h(\hat{x}(t), {}^iP_r)] \\ \dot{P} &= F[x]P + PF[x]^T + Q - PH[x, {}^iP_r]^T R^{-1}(t)H[x, {}^iP_r]P \end{aligned}$$

where

$$(15) \quad F[x] = \left. \frac{\partial f(x(t))}{\partial x} \right|_{x=\hat{x}(t)}$$

and

$$(16) \quad H[x, {}^iP_r] = \left. \frac{\partial h(x(t), {}^iP_r)}{\partial x} \right|_{x=\hat{x}(t)}$$

In this continuous time linearization,  $P(t)$  cannot be precalculated as it can if we linearize with respect to a nominal trajectory. It has to be calculated in real time, since it is coupled to the current estimate  $\hat{x}(t)$  through the linearization procedure.

It is clear from equations 9 and 16 that the measurement vector and the measurement Jacobian both depend on the position of the observer in the inertial frame. Therefore we can depict two situations

- The observer position is a function of time known a priori (including a constant). This is the “classical” case to which an extended Kalman filtering is applied.
- The observer position in the inertial frame is not known a priori but can be varied through some dynamics. This case is “non-classical”, because the observation can itself be controlled.

The location of the measurement device is then to be governed by the task: what information are we seeking? This point of view implies the definition of a criterion that reflects our objective from the gathering process. Obtaining the frequency, the position and the velocity of the target with minimum error, suggests a criteria function that depends on the FIM matrix or the state error covariance matrix  $P(t)$ . The covariance matrix  $P(t)$  provides a measure for the amount of uncertainty in the estimate of the state variables.

In order to optimize the estimation process, we define as optimization criteria the weighted trace of the state error covariance matrix. It follows that considering an observation time interval (horizon)  $[t, t + T]$  such that as  $t$  advances so does the horizon, our objective is then to minimize

$$(17) \quad R_w \text{tr}(P(t + T)) = R_w E\{(x(t + T) - \hat{x}(t + T))^T (x(t + T) - \hat{x}(t + T))\}$$

where  $x(t + T)$  presents the true state of the object at time  $t + T$  and  $\hat{x}(t + T)$  the estimated one. Note that the smaller  $\text{tr}(P(t + T))$  the more accurate is the estimation.

Obviously, the behavior of the sensor system is then entirely governed by the optimization function, and by the information available to the system through the dynamics of the object.

## 2.2 Observation policy

The definition of an optimal observation policy during the time interval  $[t, t + T]$  is based on the fact that the measurement equations are explicit functions of the receiver position. Hence, the quality of the measurements, consequently the estimates, depends on the relative location of the sensor with respect to the object. More precisely, to maximize the information content of the observations, we may vary the receiver position. The moving horizon approach was originally formulated as a method of stabilizing time varying systems, without requiring information about the system model over all future time, by minimizing a cost function up to some finite horizon ahead at each instant. The resulting controller defined on the interval  $[t, t + T]$  is only actually used for a time interval  $[t, t + t_u]$  where  $t_u < T$  before being recalculated [25].

In this study, we consider a receding horizon observation strategy with an optimization on each observation interval. We define the update time as the time to initiate a new computation of the control strategy after a certain number of measurements. The current measurement history is used in control strategy computation for a defined observation horizon. The advantage of this procedure in deriving the observation strategy is that we can deal with short term variation of the system, mainly due to the target manoeuvre error covariance matrix. The moving horizon approach can then be viewed as a good compromise since in the extended Kalman filter the error covariance matrix cannot be accurately precomputed because it is coupled to the estimation equation and an approximation on a short observation time interval is needed.

Therefore, given the derived equations of the extended Kalman filter, and given the dynamics of the receiver

$$(18) \quad \dot{x}_r(t) = V \sin(u)$$

$$(19) \quad \dot{y}_r(t) = V \cos(u)$$

$V$  defines the constant speed of the sensor. It is evident that the state vector  $\begin{bmatrix} x_r(t) & y_r(t) \end{bmatrix}$  presents the dynamics of  $iP_r$ . Obviously no command  $u(t)$  can be calculated to minimize the previous cost function because of the properties of the solution to the Riccati equation  $P(t)$  defined with respect to the estimate. Thus one approximation is to derive the extended Kalman filter with respect to a nominal trajectory which still allow to take into account the disturbances that will occur in the future.

This approach consists of the linearization of the system with respect to some nominal trajectory [12]. This nominal trajectory is defined as the predicted trajectory. The problem is then to find the command  $u(t)$  that minimize the following cost function,

$$(20) \quad J = R_w \text{tr}(P_n(t + T))$$

where  $P_n(t + T)$  is the solution of the Riccati equation derived with respect to the nominal predicted trajectory. This trajectory is defined considering the prediction of the state vector  $x(t)$ .

Given the previous equations, we can see that for a given observation approach, we have a defined estimation error covariance matrix  $P_n(t)$  solution of the Riccati differential equation and a corresponding value of the cost function. Hence, we can transform the optimization problem to a deterministic control problem [2].

## 2.3 Sub-optimal observation policy

Given the deterministic, Riccati differential equation[6]

$$\dot{P}_n(t) = F[x_n(t)]P_n(t) + P_n(t)F^T[x_n(t)] + Q - P_n(t)H^T[x_n(t), {}^iP_r]R^{-1}(t)H[x_n(t), {}^iP_r]P_n(t) \quad (21)$$

where  $H[x_n(t), {}^iP_r]$  is the Jacobian of the observation equations generated along the nominal predicted trajectory. This trajectory is defined considering the prediction of the state vector  $x(t)$ . To find the optimal  $u(t)$  such that the cost function (equation 20) is minimized and since the accuracy criteria does not include any integration, we form the following Hamiltonian function

$$(22) \quad H_h = R_w \text{tr}(\dot{P}_n(t)C_p(t)^T) + C_x \dot{x}_r(t) + C_y \dot{y}_r(t)$$

where  $C_p$  defines the costate matrix and  $\begin{bmatrix} C_x & C_y \end{bmatrix}$  the costate vector. The costates satisfy the following equations,

$$\begin{aligned} \dot{C}_p(t) &= -\frac{\partial H_h}{\partial P_n(t)} \\ &= -F[x_n(t)]^T C_p(t) - C_p(t)F[x_n(t)] \\ &\quad C_p(t)P_n^T(t)H^T[x_n(t), {}^iP_r]R^{-1}(t)H[x_n(t), {}^iP_r] + \\ &\quad H^T[x_n(t), {}^iP_r]R^{-1}(t)H[x_n(t), {}^iP_r]P(t)C_p(t) \\ \dot{C}_x(t) &= -\frac{\partial H_h}{\partial x_r(t)} \\ (23) \quad \dot{C}_y(t) &= -\frac{\partial H_h}{\partial y_r(t)} \end{aligned}$$

and the boundary conditions:

- At  $t = t$ :  $P_n(t) = P_t$ ,  $x_n(t) = \hat{x}_t$ ,  $x_r(t) = x_{r0}$ ,  $y_r(t) = y_{r0}$
- At  $t = t + T$ :  $C_p(t + T) = R_w I$ ,  $C_x(t + T) = 0$  and  $C_y(t + T) = 0$

The equations, stated above, that define the properties of the optimal observation strategy present a nonlinear two point boundary value problem where it is hard to find other than a numerical solution. Since we have nonlinear matrix differential equations, we use a variable metric technique [26] to solve the problem.

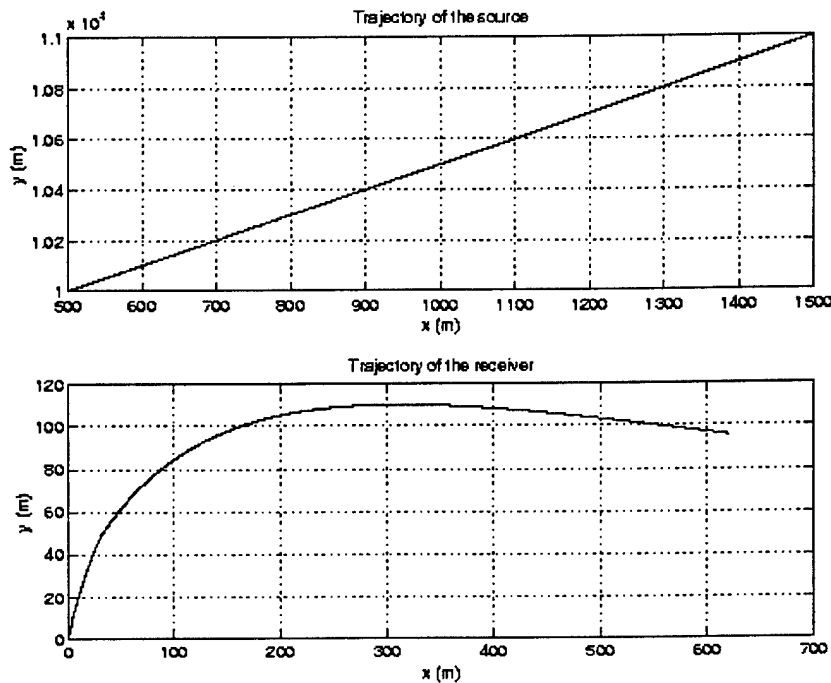


Figure 4: Emitter rectilinear trajectory

## 2.4 Simulations

The theory is verified by performing a number of simulations. The following scenario has been used to illustrate the behavior of the optimization approach and to display the measurement strategy of the optimal observer.

1. The receiver, initially located at  $(0,0)m$  in the inertial frame, moves with a constant velocity equal to  $3m/s$  and a course  $u(t)$  that has to be determined.
2. The target is initially located at  $(0.5, 10)km$  in the inertial frame. Different dynamics are considered in these simulations, mainly a constant speed  $5m/s$  rectilinear trajectory as well as a trajectory where the target is maneuvering.

The receding horizon interval is considered constant except in figure 8 where the performances of the optimization approach are tested as functions of the observation interval. Target bearing and Doppler-shift frequency are measured every 0.5 seconds. We assume a known constant emitted frequency,  $f = 3GHz$ . The measurement vector  $y$  at time  $t_i$  is required to depend on the states according to the measurement equations:

$$(24) \quad y_1 = \arctan \frac{x_r(t_i) - x(t_i)}{y_r(t_i) - y(t_i)}$$

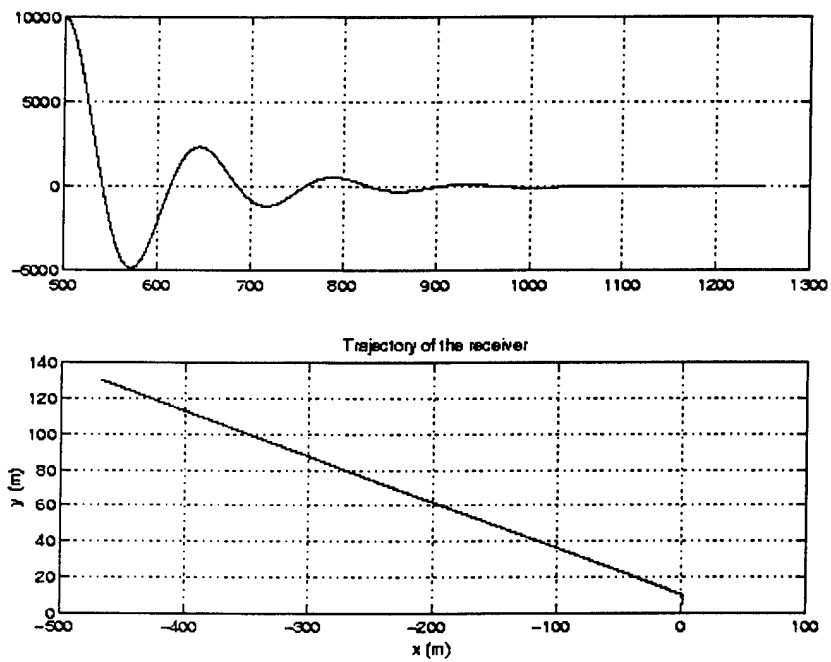


Figure 5: Emitter manoeuvre trajectory

$$\begin{aligned}
 y_2 = f \{ & 1 + \frac{1}{c\sqrt{(x_r(t_i) - x(t_i))^2 + (y_r(t_i) - y(t_i))^2}} [ \\
 & (V \sin(u(t_i)) - V_x)(x_r(t_i) - x(t_i)) + \\
 & (V \cos(u(t_i)) - V_y)(y_r(t_i) - y(t_i))] \}
 \end{aligned}
 \tag{25}$$

The simulation results are presented in figures 4, 5, 6, 7, and 8. Figures 4 and 5 show the trajectories of the emitter and the receiver in the inertial frame. It is clear from the results that the receiver is maneuvering more when the emitter is following a rectilinear trajectory and vice versa. This kind of maneuver is closely related to the convergence of the filter by not allowing it to close up very fast, consequently allowing to the state estimates to converge to the real values. Figure 6 shows the errors in the estimates. In this figure, we also present the results obtained in the case where we measure only the bearing angle with a controlled receiver position. It is evident from the results that considering both the bearing angle and the Doppler-shift frequency outperform the bearing measurement approach. It is important to note here that in the initialization of the optimization process, we did not resort to defining different legs for the course of the receiver [19], depending on the expected dynamics of the source, which make the proposed optimization approach more straightforward to use. Figure 7 presents the trajectories of the receiver for both bearing only measurements and bearing and Doppler-shift frequency measurements. It is clear that for the same emitter dynamics, the receiver has to maneuver in the bearing only scenario. Adding the Doppler-shift frequency to the measurement vector allows a smoother trajectory for the receiver and a better quality of the estimates. Figure 8 presents the effects of varying the receding horizon interval on the quality of the estimates, we limit the plots to the estimation error in the  $y$  direction. It is evident that the length of the interval influence the accuracy of the estimates. The choice of this interval is mainly governed by the dynamics of the target and the sought accuracy of the estimates.

Simulations suggest that the measurement technique presents satisfactory results for the specification of the optimal measurement strategy. Even though we use a suboptimal approach and we linearize with respect to a predicted nominal trajectory given the fact that when using an extended Kalman the covariance matrix is coupled to the estimation equation, the proposed technique allows for the tracking of an object with only a predicted motion.

### 3. Determination of active and passive measurements strategies

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Many tracking systems involve basically active and passive subsystems. However there are also physical constraints such as field of view, environment, cost and risk of the operation that impose the requirement that, at each instant of time, a subset of the set of



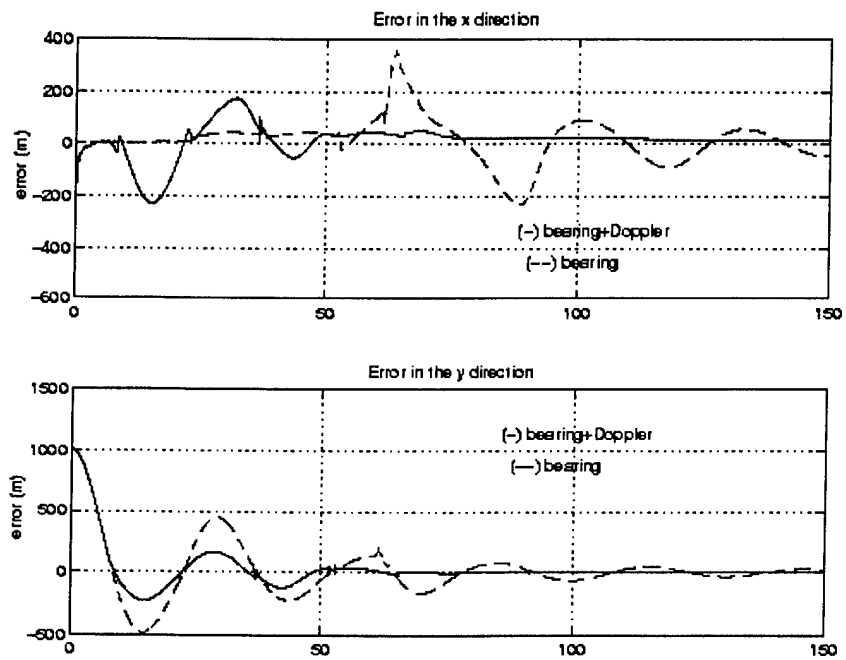


Figure 6: The estimates errors

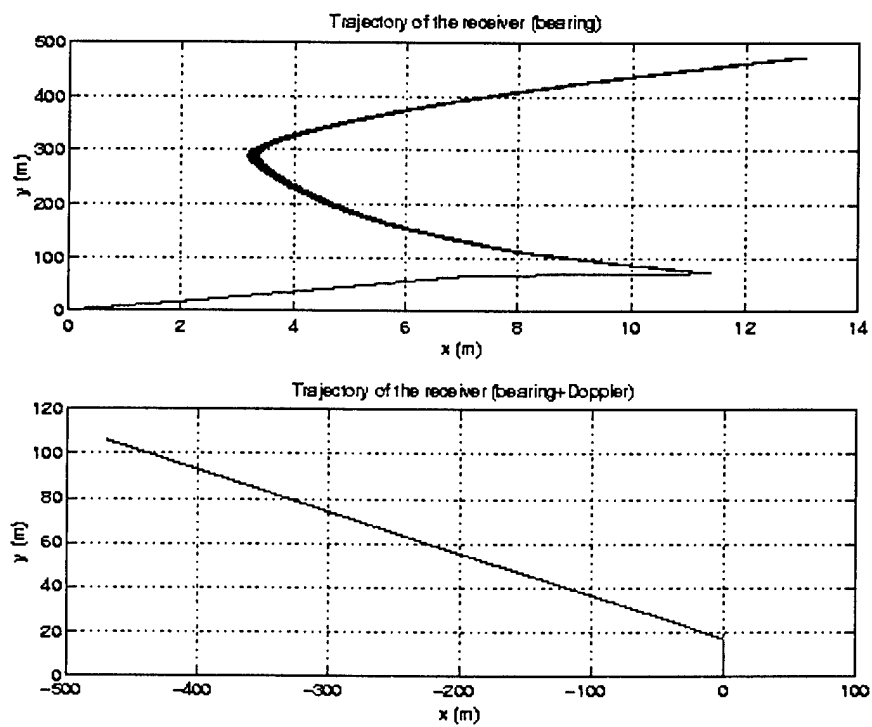


Figure 7: Trajectory of the receiver as a function of the considered measurements

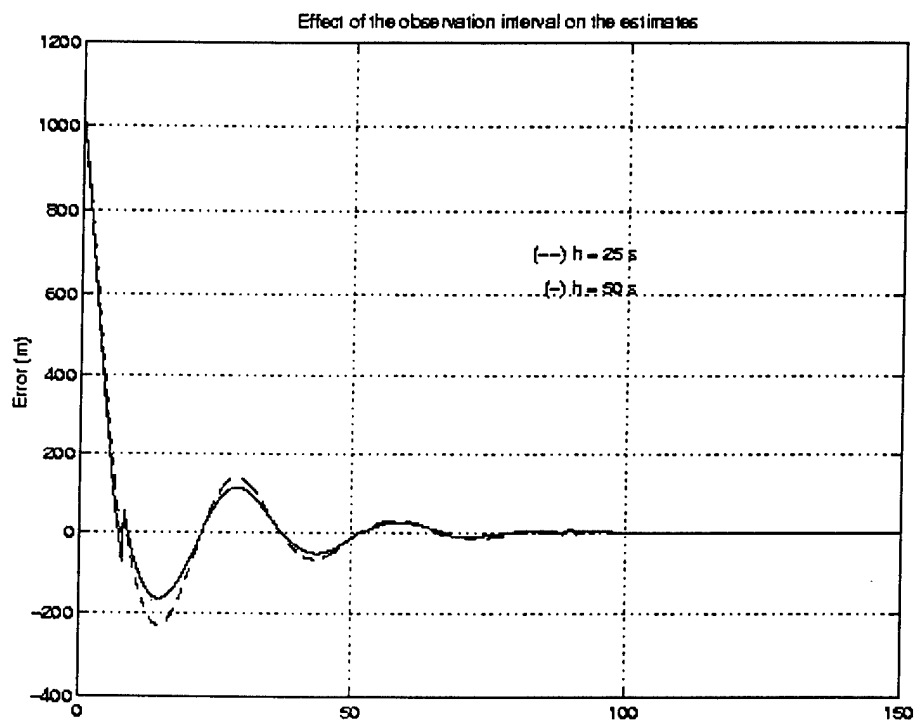


Figure 8: Estimates errors as a function of the observation receding horizon interval

sensors is used. In such cases, one has to make a decision: which measurements to make at present, and when to make alternate measurements. The problem of measurement scheduling has a long history [9, 3, 11]. Most of these studies were limited to linear stochastic systems. In recent years, somewhat related problems have been studied [4, 13]. In these studies, the radar allocation logic was presented and different parameters have been used. In this study, we consider an extension of [3] to a nonlinear system where an alternate observation policy is presented.

### 3.1 Problem Formulation

To establish a dynamic model for the target motion, the following assumption is made in advance: our purpose is to estimate, rather than regulate, the target motion, so the resulting model is not necessarily controllable.

#### 3.1.1 Target dynamics

The dynamic model for the target is assumed to be:

$$(26) \quad \dot{x}(t) = f(x(t)) + \xi(t) \quad x(0) = x_0$$

It is a time varying system describing the position and the motion of the object in an inertial frame. We assume that the system state vector  $x(t)$  is an  $n$  dimensional column vector. The system driving noise  $\xi(t)$  is assumed to be a Gaussian white noise  $N(0, Q)$  with zero mean and a fixed covariance matrix. The initial state vector  $x_0$  is modelled as a random vector with known mean,  $\hat{x}_0$ , and known covariance  $P_0$ . The dynamic model of the target motion may be as simple as  $\dot{x}(t) = Ax(t) + \xi(t)$ , where  $A$  is known, and as complex as a model for a ballistic missile.

#### 3.1.2 Sensor constraints

Assuming that we have  $M$  basic sensors  $s_1, \dots, s_M$  numbered from 1 to  $M$ . The pseudo sensors, comprised of the combinations of the basic sensors, can be numbered from  $M + 1$  up to  $2^M - 1$  [18]. We shall let  $y_j(t)$  denote the measurement vector obtained from the  $j$ -th sensor, at time  $t$ . The size of the measurements vector depends on the number of basic sensors used and the number of targets in view. For simplicity, we assume one source target in the field of view of the observer. The vector of measurements  $z_j(t)$  is a variable dimension vector given by:

$$z_j(t) = y_j(t) + \nu_j(t), j = 1, 2, \dots, 2^M - 1$$

$\nu_j(t)$  is the measurements noise. For simplicity we assume that the basic sensors measurements noises are independent, Gaussian white noises with zero means and constant, known, positive definite, covariance matrices  $R$ . It follows that the pseudo sensor measurement noise is also a Gaussian white signal  $\nu_j(t)(0, R_j)$ . We assume that each noise process is independent of  $x_0$  and  $\xi(t)$  for all  $t \geq t_0$ , all  $j = 1, 2, \dots, 2^M - 1$ . At each instant of time  $t$ , we are constrained in looking at only the output of a sensor  $j$ , a basic sensor or a pseudo sensor but we are able, considering certain constraints to switch from one sensor to another. A convenient way of modelling this sensor selector [3] is to define  $2^M - 1$  time functions, denoted by

$$(27) \quad v_1(t), \dots, v_j(t), \dots, v_{2^M-1}(t)$$

with the following properties

- at each instant of time  $v_j(t)$  can have the value 0 or 1
- if  $v_j(t) = 1$  then  $v_k(t) = 0$  for  $k = 1, \dots, j-1, j+1, \dots, 2^M - 1$ .

The measurement vector at instant  $t$  can then be written as

$$(28) \quad z(t) = v_1(t)z_1(t) + v_2(t)z_2(t) + \dots + v_{2^M-1}(t)z_{2^M-1}(t)$$

with  $\dim(z(t)) = \dim(z_j(t))$  when  $v_j(t) = 1$

### 3.1.3 Observations cost

We can associate an observation cost to each one of the sensor. Such a cost can be used to reflect that special basic sensors may be required to carry out a specific observation or special time requirements are considered in making observations. For this reason, we assume that there is an inherent cost that must be taken into account in order to arrive at an optimal measurement policy. We denote by  $q_j(t)$  the per-unit-of time cost of making the measurement  $z_j(t)$  at time  $t$ . Since one is limited to a specific sensor at each instant of time, then one can associate with each observation policy a total cost  $q(v)$  defined by

$$(29) \quad q(v) = \int_{t_0}^T \left[ \sum_{j=1}^{2^M-1} q_j(t)v_j(t) \right] dt$$

$q(v)$  represents the total observation cost associated with the use of the observation strategy in the time interval  $[t, t + T]$

The definition of an optimal observation strategy cannot be based only on the observation cost. Assuming that the purpose of the measurements is a TMA defined by the prediction of the target dynamics, intuitively, one would expect

that the accuracy of any prediction will depend on the information content and accuracy of the measurements that have been already made. Hence, an optimal observation policy must depend, in addition to the cost of observation, upon the accuracy of the prediction for which observations are made.

In this study, we assume that the purpose of the observation during the time interval, observation horizon,  $[t, t + T]$  is the prediction of the target dynamics  $x(T_p)$ ,  $T \leq T_p$ , where  $T_p - T$  is the length of the prediction interval. Assuming that  $\hat{x}(T_p)$  is the predicted state, it follows that the accuracy of the prediction is defined by

$$(30) \quad J(T_p) = E\{(x(T_p) - \hat{x}(T_p))^T(x(T_p) - \hat{x}(T_p))\}$$

Note that the smaller  $J(T_p)$ , the more accurate is the prediction.

### 3.2 Optimization Approach

In this study and for simplicity we consider linear dynamics for the target

$$(31) \quad \dot{x}(t) = Ax(t) + \xi(t) \quad x(0) = x_0$$

and given the sensor measurements

$$z_j(t) = y_j(t) + \nu_j(t); j = 1, 2, \dots, 2^M - 1$$

Let  $[t, t + T]$  the receding horizon observation interval and  $t + T_p$  the prediction time. Determine the scalar variables

$$(32) \quad v_1(t), v_2(t), \dots, v_{2^M-1}(t); t \in [t, t + T]$$

subject to the constraints

$$(33) \quad v_j(t) \in \{0, 1\}; \sum_{j=1}^{2^M-1} v_j(t) = 1$$

and the scalar cost functional [3]

$$(34) \quad J = \alpha \int_t^{t+T} \left[ \sum_{j=1}^{2^M-1} q_j(t) v_j(t) \right] dt + E\{(x(t+T_p) - \hat{x}(t+T_p))^T(x(t+T_p) - \hat{x}(t+T_p))\}$$

where  $\alpha$  defines the weighting on the observations cost.

A useful formalism for computing the estimates is the Kalman filter. One of the most compelling arguments for using this framework is its mathematical clarity. Furthermore we believe that:

- In the situations we are considering, some prior information is available.

- We have a well justified basis for the chosen cost functional.
- It defines a powerful tool to maximize the speed of convergence of the estimates to true values by application of advance knowledge.

For this and because of the nonlinearities of the models, we consider an extended Kalman filter to derive the estimates. The basic idea of the extended Kalman filter is to relinearize the sensor and dynamic models about each predicted estimate once it has been computed. As soon as a new state estimate is made, a new and better reference state trajectory is incorporated into the estimation process. Thus allowing the validity of the assumption that deviations from the nominal trajectory are small enough to allow linear perturbation techniques. Under the assumption of continuous time process, continuous time measurements, the extended Kalman filter cycle is given by the following equations:

$$(35) \quad \dot{\hat{x}}(t) = A\hat{x}(t) + P(t) \left[ \sum_{j=1}^{2^M-1} v_j(t) H_j(t) R_j^{-1} \right] \left[ z(t) - \left( \sum_{j=1}^{2^M-1} v_j(t) H_j(t) \right) \hat{x}(t) \right]$$

The error covariance matrix  $P(t)$  is the solution of the matrix Riccati differential equation

$$(36) \quad \dot{P}(t) = AP(t) + P(t)A^T + Q - P(t) \left[ \sum_{j=1}^{2^M-1} v_j(t) H_j^T(t) R_j^{-1} H_j(t) \right] P(t)$$

where

$$(37) \quad H_j(t) = \frac{\partial y_j(x(t))}{\partial x} \Big|_{x=\hat{x}(t)}$$

In this continuous time linearization,  $P$  cannot be precalculated as it can if we linearize with respect to a nominal trajectory. It has to be calculated in real time, since it is coupled to the current estimate  $\hat{x}(t)$  through the linearization procedure.

The predicted estimate  $\hat{x}(t + T_p)$  of the state  $x(t + T_p)$  can be computed from the state estimate  $\hat{x}(t + T)$  by

$$(38) \quad \hat{x}(t + T_p) = \Phi(t + T_p, t + T) \hat{x}(t + T)$$

$\Phi(t, \tau)$  defines the transition matrix

$$(39) \quad \dot{\Phi}(t, \tau) = A\Phi(t, \tau); \Phi(\tau, \tau) = I$$

It follows that the cost functional to minimize is given by

$$J = \alpha \int_t^{t+T} \left[ \sum_{j=1}^{2^M-1} q_j(t) v_j(t) \right] dt + tr [\Phi(t + T_p, t + T) P(t + T) \Phi^T(t + T_p, t + T)] \quad (40)$$

We remark that this is a deterministic optimal control problem. Obviously no measurement strategy can be calculated to minimize the previous cost functional because of the properties of the solution to the Riccati equation  $P(t)$  defined with respect to the estimate. Thus one approximation is to derive the extended Kalman filter with respect to a nominal trajectory which still allow to take into account the disturbances that will occur in the future. This approach consists of the linearization of the system with respect to some nominal trajectory [12]. This nominal trajectory is the predicted trajectory. The problem is then to find the optimal observation policy  $v_j(t)$  that minimizes

$$J = \alpha \int_t^{t+T} \left[ \sum_{j=1}^{2^M-1} q_j(t) v_j(t) \right] dt + tr [\Phi(t + T_p, t + T) P_n(t + T) \Phi^T(t + T_p, t + T)] \quad (41)$$

In this study, we consider a receding horizon observation strategy with an optimization on each observation interval. We define the update time as the time to initiate a new computation of the observation strategy after a certain number of measurements. The advantage of this procedure in deriving the observation strategy is that we can deal with short term variation of the system. Since the dynamic constraints are naturally expressed via a matrix differential equation, one can obtain the solution through the use of the matrix minimum principle [2].

### 3.3 Reformulation of the optimization Approach

Given the deterministic, Riccati differential equation

$$\dot{P}_n(t) = AP_n(t) + P_n(t)A^T + Q - P_n(t) \left[ \sum_{j=1}^{2^M-1} v_j(t) H n_j^T(t) R_j^{-1} H n_j(t) \right] P_n(t)$$

where  $H n_j(t)$  is the Jacobian of the observation equation generated along the nominal predicted trajectory. Let  $G(t)$  denote an  $n \times n$  costate matrix associated with the covariance matrix  $P_n(t)$ . We define the scalar Hamiltonian function for the posed optimization problem as follows

$$(42) \quad H_h = \alpha \sum_{j=1}^{2^M-1} q_j(t) v_j(t) + tr \dot{P}_n(t) G^T(t)$$

or



$$\begin{aligned}
H_h = & \alpha \sum_{j=1}^{2^M-1} q_j(t)v_j(t) + tr [AP_n(t)G^T(t)] + tr [P_n(t)A^TG^T(t)] + \\
(43) \quad & tr [QG^T(t)] + tr \left[ P_n(t) \left( \sum_{j=1}^{2^M-1} v_j(t)Hn_j^T R_j^{-1} Hn_j \right) P_n(t)G^T(t) \right]
\end{aligned}$$

Assuming that  $v_j^*(t)$  characterize the optimal observation strategy,  $P_n^*(t)$  the resultant state error covariance matrix, and  $G^*(t)$  the corresponding costate matrix. Then the following properties are true

$$\begin{aligned}
\dot{G}^*(t) = & -\frac{\partial H_h}{\partial P_n^*(t)} \\
= & -A^T G^*(t) - G^*(t)A \\
& + G^*(t)P_n^{*T}(t) \left( \sum_{j=1}^{2^M-1} v_j^*(t)Hn_j^T(t)R_j^{-1}Hn_j(t) \right) \\
& + \left( \sum_{j=1}^{2^M-1} v_j^*(t)Hn_j^T(t)R_j^{-1}Hn_j(t) \right) P_n^*(t)G(t) \\
(44) \quad & \\
\dot{P}_n^*(t) = & \frac{\partial H_h}{\partial G(t)} \\
= & AP_n^*(t) + P_n^*(t)A^T + Q - P_n^*(t) \left[ \sum_{j=1}^{2^M-1} v_j(t)Hn_j^T(t)R_j^{-1}Hn_j(t) \right] P_n^*(t) \\
(45) \quad &
\end{aligned}$$

under the following boundary conditions:

- At  $t = t$ :  $P_n^*(t) = P_t$ ,  $x_n(t) = \hat{x}_t$
- At  $t = t + T$ :  $G^*(t + T) = \frac{\partial H_h}{\partial P_n(t+T)}$   
 $= \Phi^T(t + T_p, t + T)\Phi(t + T_p, t + T)$

The equations, stated above, that define the properties of the optimal observation strategy present a nonlinear two point boundary value problem. A technique which can be used, is the min-H technique [3, 8]

### 3.4 Simulations

A simple scenario has been used to illustrate the behavior of the optimization approach and to display the measurements strategy. The scenario is the following: we consider

one observer with a collocated radar and ESM basic sensors. The receiver trajectory is defined by

$$\begin{aligned}
 x_r(t) &= r \cos(2\pi f_c t) \\
 y_r(t) &= r \sin(2\pi f_c t) \\
 z_r(t) &= cste
 \end{aligned}
 \tag{46}$$

where  $r$  is the radius of the circle flown by the receiver, and  $T_c = 1/f_c$  is the time it takes to complete one flight around the circle. Only one target is in the scene. Initially located at  $(5e2, 10e3)m$  in the inertial frame, the target moves with a constant velocity equal to  $5m/s$  in each direction. The relative target bearing, elevation and, range, with respect to the observer, are measured every 0.5 seconds. The receding horizon is chosen equal to  $50s$ . The measurement vector  $y$  at time  $t_i$  is required to depend on the states according to the measurement equations:

$$\begin{aligned}
 y_1 &= \arctan\left(\frac{x_r(t_i) - x(t_i)}{y_r(t_i) - y(t_i)}\right) \\
 y_2 &= \arctan\left(\frac{z_r(t_i) - z(t_i)}{\sqrt{(x_r(t_i) - x(t_i))^2 + (y_r(t_i) - y(t_i))^2}}\right) \\
 y_3 &= \sqrt{(x_r(t_i) - x(t_i))^2 + (y_r(t_i) - y(t_i))^2 + (z_r(t_i) - z(t_i))^2}
 \end{aligned}$$

The radar measurements are defined by  $y_2, y_3$ . The ESM measurements are defined by  $y_1$  and the pseudo-sensor measurements by  $y_1, y_2, y_3$ .

In this simulation, the trajectory of the observer is chosen so as to ensure the observability of the target based on the measurements of only one sensor. It follows that the observation policy is not directly governed by the observability of the target but by the properties of the sensors and the observation interval. The target parameters to estimate are collected in a four-dimensional vector  $x(t) = [x, y, \dot{x}, \dot{y}]$ . For simplicity, we assume a known target elevation.

The simulation results are presented in figures 9, 10, 11, and 12. Figures 9 and 10, present the observation policy when varying the measurements noise of both sensors. In figure 9, we consider that the ESM and the radar noises have an equal variance for the angle measurements  $\sigma = 1degree$ . In figure 10, The ESM sensor has  $\sigma = 10degree$ . It is clear from the obtained results that without an additional cost on the use of a specific sensor  $\alpha = 1$  and  $q_j(t) = 1$ , the observation strategy relies on both sensors if they have similar resolutions. Otherwise, figure 2, the observation strategy will rely on the more precise sensor for a while before using the two sensors. Figures 11 and 12 present the estimations errors for the described scenarios. It is evident that the ESM properties do not affect the quality of the estimates.

As with all nonlinear two-point boundary value problems, there is no guarantee of convergence to a global optimum and depending on the initial guess the algorithm would sometimes oscillate between two observation strategies [3]. Despite this, the

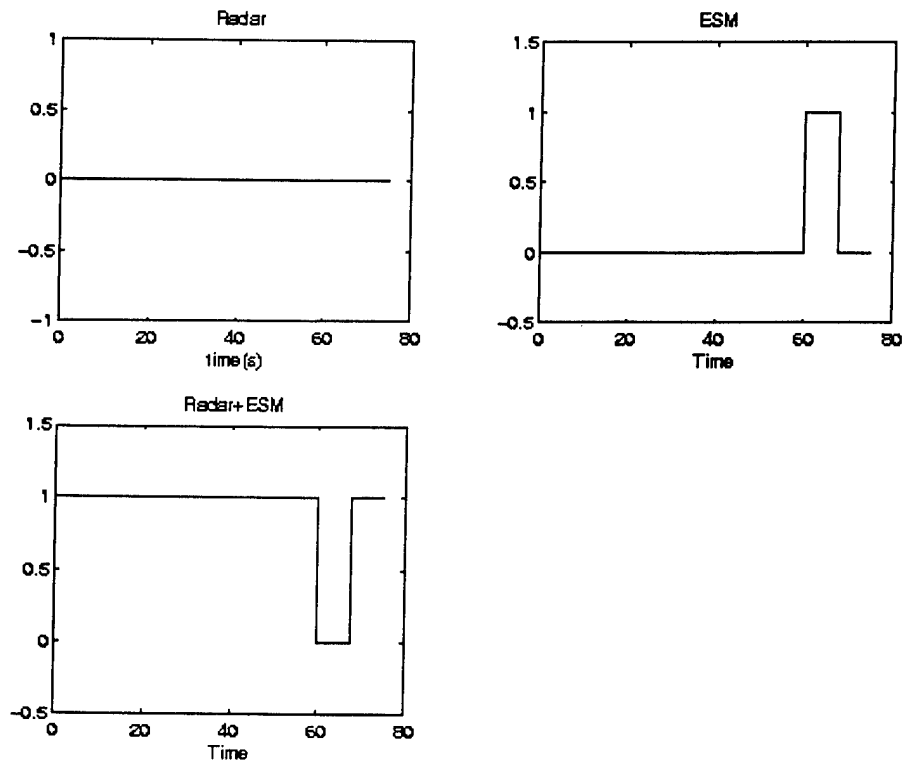


Figure 9: Measurements strategy for the sensors  $R_{esm} = 1$

approach is simple to implement and the obtained results indicate an optimal approach at least for the intuitive example considered in the simulations.

In this study, an algorithm for sub-optimal sensor selection has been proposed and simulation results suggest that this technique can be used to manage multiple sensors. This algorithm is based on an optimization approach that takes into account the properties of the sensors and the objective of the measurements. These parameters are integrated in the definition of the cost functional. Results outline that the measurement strategy depends on the characteristics of the sensors, the observation interval and the prediction interval. Although we use a suboptimal approach, we linearize with respect to a predicted nominal trajectory, and we use a min-H algorithm, the approach is simple and the obtained results are promising.

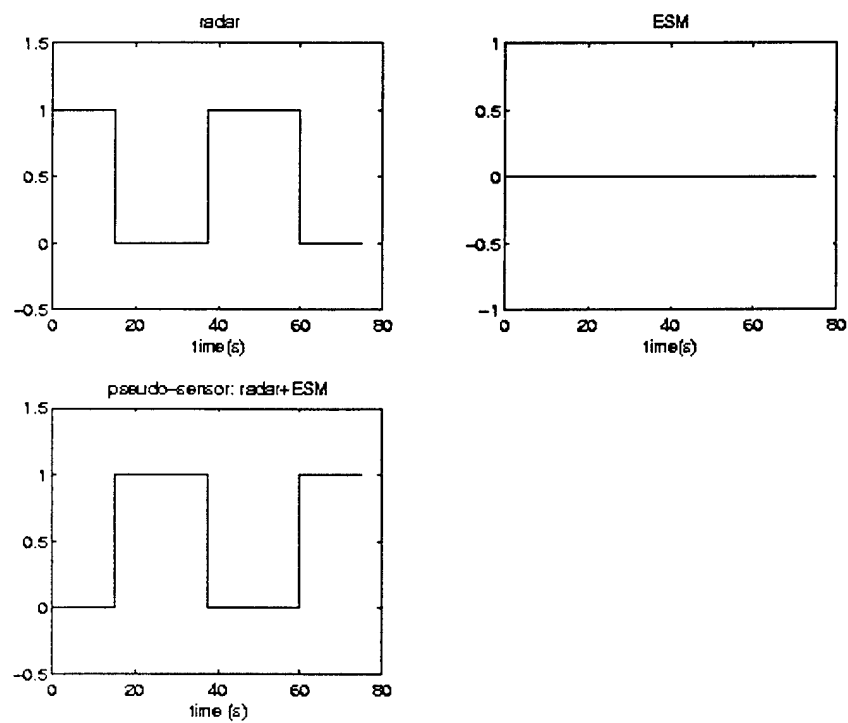


Figure 10: Measurements strategy for the sensors  $R_{esm} = 10$

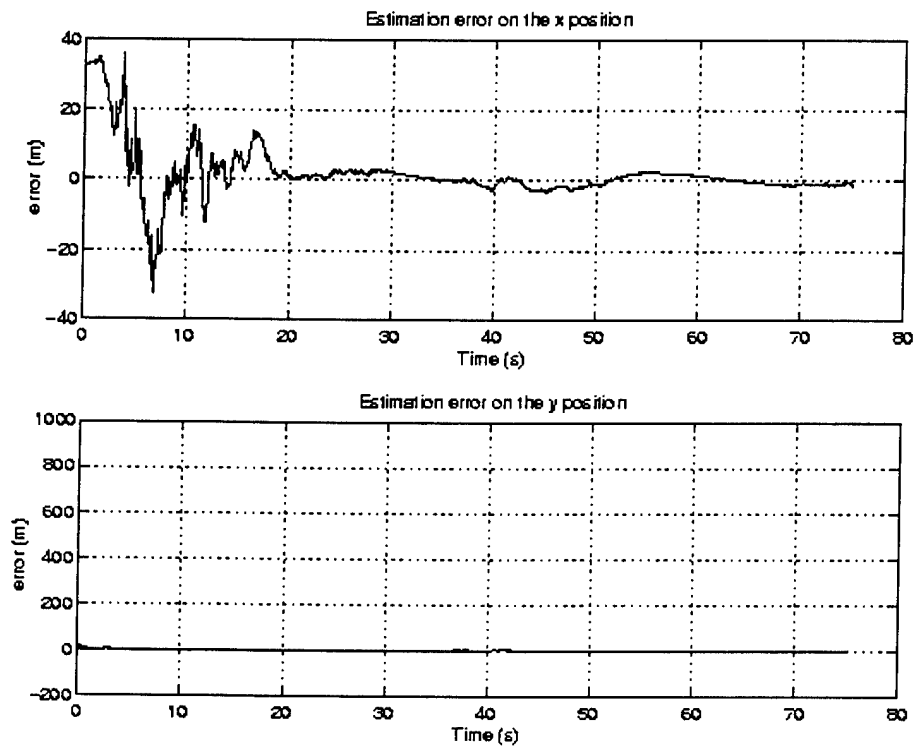


Figure 11: Estimation Errors for  $R_{estm} = 1$

## 4. Conclusion and the way ahead

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This report presented two comprehensive management cases where the benefits of fusion and sensor management are made evident. In the first case, the position of a passive sensor is considered as a manageable parameter. In the second case, a measurement coordination problem between a passive sensor and an active one is presented.

Simulations suggest that the measurement technique, considered in the first study case, presents satisfactory results for the specification of the optimal measurement strategy. Even though a suboptimal approach is used, the proposed technique allows for the tracking of an object with only a predicted motion.

In the second case, an algorithm for sub-optimal sensor selection has been proposed and simulation results suggest that this technique can be used to manage multiple sensors. This algorithm is based on an optimization approach that takes into account the properties of the sensors and the objective of the measurements. These parameters are integrated in the definition of the cost functional. Results outline that the measurement strategy depends on the characteristics of the sensors, the observation interval and the prediction interval. Although a suboptimal approach is used, the obtained results are promising.

From this study, it is evident that the prominent role of data fusion and of sensor management, as well, is one in which all sensors are exploited to solve state estimation/prediction task. In general, however, the lack of standardized performance evaluation, system engineering methodologies, architecture paradigms, or multi-spectral models of targets and collection systems has been a major impediment to integration and adaptive fusion. In short, current developments do not lend themselves to objective evaluation, comparison or re-use. Often, the role of data fusion has been unduly restricted to a subset of the processes and relevant to particular state estimation problems. For example, in military applications such as targeting or tactical intelligence, the focus is on estimating and predicting the state of specific types of entities in an external environment. In this context, the applicable sensors/sources sequences are already defined by the system designer.

Ultimately, however, such problems are inseparable from problems of navigation, of calibrating sensor alignment and performance, and of validating one's library of target models. A more powerful realization of the role of data fusion and, indeed, of resource management as well, is one in which all sources are exploited to solve all required tasks problems. In this realization, the coupling of data fusion tree with resource management tree is an interactive operation and the design of both trees will play an important part in effective system designs. Developing data fusion functionality with an

information processing system can be based on the following four phases as suggested by Steinberg [28]:

1. **Operational Architecture Design:** System-level problem decomposition; assigning the role for data fusion, as well as for other system functions (sensors, communications, response resources, human operators,.....etc.)
2. **System Architecture Design:** Design of the data fusion tree by partitioning the process among C3 nodes and into processing nodes; specifying interaction with sensors/sources, resource management nodes, and information users.
3. **Component Function Design:** Design of data fusion nodes, to include specifying data inputs/outputs of component functions (alignment, association and estimation), allocation to human/automatic processes, and technique selection.
4. **Detailed Design and Development:** Pattern application, algorithm tailoring, software adaptation and development.

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This report addresses the problem of data fusion and sensor management based on a synergistic use of the information provided by multiple sensors. In the first studied case, we deal with the problem of measurement strategy computation for a passive receiver. The basic problem is then to compute an optimal policy, during a specified observation time interval so that a prediction accuracy is optimized. It is shown that the optimal measurement policy can be precomputed before the measurements actually occur. The second case describes research work on the selection of a strategy of measurements for an active and a passive systems. The approach is based on selecting at each instant of time, a set of measurements provided by one or more sensors. Each sensor measurement has an associated cost. The basic problem is then to select an optimal measurement policy, during a specified receding horizon observation interval, so that a weighted combination of prediction accuracy and observation cost is optimized.

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